**2.Methodology**

The Black-Litterman(BL) asset allocation model, created by Fisher Black and Robert Litterman, is a sophisticated portfolio construction method that overcomes the problem of unintuitive, high-concentrated portfolios, input-sensitivity, and estimation error maximization. It is an asset allocation model which bases on the mean-variance optimization model and capital asset pricing model (CAPM). The things which differ in BL model is the way which we are observing expected asset returns which is one of the inputs into the forward optimizer. Following two figures show these two optimization processes:

**Figure 1** Mean-Variance optimization process

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**Figure 2** Black-Litterman optimization process

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**2.1 Reverse Optimization**

Instead of using the historical asset return, BL use the equilibrium asset returns. Equilibrium asset returns are returns implied from the market capitalization weights of individual assets or asset classes. It can be derived by a reverse optimization method in which the vector of implied excess equilibrium using the formula:

Where

is the Implied Excess Equilibrium Return Vector (N x 1 column vector);

is the risk aversion coefficient;

is the covariance matrix of excess returns (N x N matrix);

is the market capitalization weight (N x 1 column vector) of the assets.

**2.2 Black-Litterman Model**

**2.2.1 The Black-Litterman Formula**

The formula for the new Combined Return Vector (E[R]) is

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Where

E[R] is the new(posterior) Combined Return Vector (N x 1 column vector);

is a scalar;

is the covariance matrix of excess returns (N x N matrix);

P is a matrix that identifies the assets involved in the views (K x N matrix or 1 x N row vector in the special case of 1 view);

Ω is a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view (K x K matrix);

Π is the Implied Equilibrium Return Vector (N x 1 column vector); and,

Q is the View Vector (k x 1 column vector).

**2.2.2 Calculating the New Combined Return Vector**

Our focus in the estimation is the combination of conditional equilibrium returns with the views using Bayesian approach. The process of combining the subjective views of an investor regarding to the expected returns of one or more assets with the market equilibrium vector of expected return is described in the Figure (3).

**Figure 3** Deriving the New Combined Return Vector (E[R])

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**2.3 Risk Adjusted Black-Litterman Portfolios**

We discussed the reversed optimizer above and following is the discussion of the forward optimizer.

2.3.1 Sharp Ratio Maximazation

In the method we use mean variance optimization and maximize the Sharpe ratio.

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Where

is the expected excess return of the BL portfolio E[R]

Σ is the conditional portfolio variance

is the N x 1 vector of portfolio weights

Thus, the vector of optimal weights for the SR-BL portfolio is given by

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